Introduction to Control of port-Hamiltonian systems

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Stabilization of PHS

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Doctoral course, Université Franche-Comté, Besançon, France

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Outline

Some basic notions on stability

We are considering the following class of state space model

$$\begin{aligned} \dot{x}(t) &= f(x(t)), \\ x(0) &= x_0, \end{aligned} \tag{1}$$

with $t \in \mathbb{R}$ and $x \in \mathbb{R}^n$. Furthermore $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$. It is assumed that $f(x(t))$ satisfies the standard assumptions for existence and uniqueness of solutions, i.e., that $f(x(t))$ is Lipschitz continuous with respect to x, uniformly in t, and piecewise continuous in t.

We want to analyse the dynamics of the system

- Do the solutions of (1) remain bounded in time?
- If yes, do they in addition converge to some *equilibrium point*?
- If yes, can we say anything of the speed of convergence?
- Can we modify the solution with some external control input to impose a desired closed-loop behaviour?

Some basic notions on stability

We are considering the following class of state space model

$$\begin{aligned} \dot{x}(t) &= f(x(t)), \\ x(0) &= x_0, \end{aligned} \tag{2}$$

with $t \in \mathbb{R}$ and $x \in \mathbb{R}^n$. Furthermore $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$. It is assumed that $f(x(t))$ satisfies the standard assumptions for existence and uniqueness of solutions, i.e., that $f(x(t))$ is Lipschitz continuous with respect to x, uniformly in t, and piecewise continuous in t.

Equilibrium position

An equilibrium point x^* is a solution to

$$\dot{x}(x^*) = 0, \qquad \text{i.e.,} \qquad f(x^*) = 0$$

Without loss of generality we will assume that $x^* = 0$.

Some basic notions on stability

An equilibrium position $x = 0$ of system (1) is

1. Stable if for any $\epsilon > 0$ and $t_0 \geq 0$, there exists a $\delta(\epsilon, t_0) > 0$ such that $\|x_0\| < \delta$ implies $\|x(t, x_0)\| < \epsilon$ for all $t \geq t_0$,
2. Uniformly Stable if δ does not depend on t_0,
3. Asymptotically Stable if it is stable and for any $t_0 \geq 0$ there exists $\Delta(t_0) > 0$ such that every solution $x(t, x_0)$ for which $\|x_0\| < \Delta$ satisfies the relation
$$\lim_{t\to\infty} \|x(t, x_0)\| \to 0 \tag{3}$$
4. Uniformly Asymptotically Stable if it is uniformly stable, Δ does not depend on t_0, and (3) holds uniformly with respect to t_0 and x_0 in the domain $t_0 \geq 0$, $\|x_0\| < \Delta$,

Some notions on stability

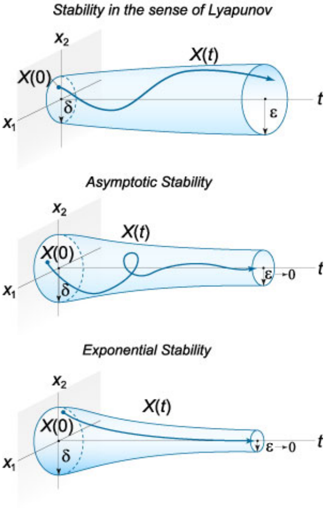
An equilibrium position $x = 0$ of system (1) is

1. Globally Asymptotically Stable if it is stable and relation (3) holds for any $t_0 \geq 0$ and x_0,
2. Uniformly Globally Asymptotically Stable if it is uniformly stable and relation (3) holds for any $t_0 \geq 0$ and x_0 uniformly relative to t_0 and x_0 in the domain $t_0 \geq 0$, $x_0 \in K$, where K is arbitrary compact in the x-space,
3. Exponentially Asymptotically Stable if there exist positive constants Δ, M, and α such that every solution $x(t, x_0)$, for which $\|x_0\| < \Delta$, satisfies the relation

$$\|x(t, x0)\| < M\|x_0\|e^{-\alpha(t-t_0)} \tag{4}$$

   for all $t \geq t_0 \geq 0$, and
4. Globally Exponentially Asymptotically Stable if there exist positive constants M and α such that relation (4) holds for $t \geq t_0 \geq 0$ and arbitrary $t_0 \geq 0$ and x_0.

Some notions on stability



Figures taken from:
http://www.math24.net/

Some notions on stability

How do we analyse stability?

Lyapunov stability theory

- Lyapunov's direct method (second method) $\rightarrow$ non-linear systems
- Lyapunov's indirect method (first method) $\rightarrow$ linear systems

Lyapunov's direct method allows to determine the stability of a system without explicitly integrating the differential equations. The method is a generalization of the idea that if there is some "measure of energy" in a system, then we can study the rate of change of the energy of the system to ascertain stability.

Some notions on stability

Let B_ϵ be a ball of size ϵ around the origin, $B_\epsilon = x \in \mathbb{R}^n : x < \|\epsilon\|$.

Positive definite function

A continuous function $V : \mathbb{R}^n \to \mathbb{R}$ is a locally positive definite function if $V(0) = 0$ and for $x \in B_\epsilon$, $x \neq 0 \to V(x) > 0$. If B_ϵ is the whole state space, then $V(x)$ is globally positive definite.

A positive definite function is like an energy function.

Lyapunov's direct method

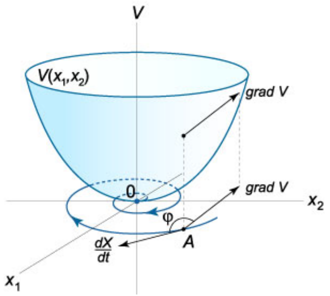
Theorem: Lyapunov stability Let $V(x)$ be a non-negative function with continuous partial derivatives such that

- $V(x)$ is positive definite on B_ϵ, and $\dot{V} \leq 0$ locally in x and for all t, then the origin of the system is locally stable (in the sense of Lyapunov).
- If in addition $V(x) \to \infty$ when $\|x\| \to \infty$, then the system is globally stable.

Theorem: Asymptotic stability Let $V(x)$ be a non-negative function with continuous partial derivatives such that

- $V(x)$ is positive definite on B_ϵ, and $\dot{V} < 0, \; \forall x \in B_\epsilon/\{0\}$ and $V(0) = 0$ locally in x and for all t, and then the origin of the system is locally asymptotically stable.
- If in addition $V(x) \to \infty$ when $\|x\| \to \infty$, then the system is globally asymptotically stable.

Lyapunov’s direct method



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For physical systems: relate the physical energy with Lyapunov functions

Stabilization of passive systems

Let us consider systems arising from some physical energy model. We then usually have

$$H(t) = H(t_0) + \underbrace{\int_0^t u(\tau)y(\tau)d\tau}_{\text{supplied energy}} - \underbrace{\int_0^t S(x(\tau))d\tau}_{\text{dissipated energy}} .$$

So if $H(x)$ qualifies as a Lyapunov function and $S(x)$ vanishes at $x = 0$ (and only in $x = 0$), then the system is asymptotically stable!

So why do we need the control then?

Stabilization of passive systems

- What if we want to increase the rate of convergence?: damping injection,
- What if we want to stabilize at some different equilibrium point, $x = x^*$, $x^* \neq 0$: Energy shaping,
- What if $S(x)$ vanishes for some $x \neq 0$ or $S(x) = 0$?: damping injection + Energy shaping

PHS are dissipative and passive systems

Consider the system

$$\dot{x}(t) = f(x(t), u(t)), \qquad y(t) = h(x(t), u(t)), \tag{5}$$

with $t \in \mathbb{R}$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$. Furthermore $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $h : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$. Let us addition define the **supply rate** $w(t) = w(u(t), y(t))$,

$$\int_0^t |w(u(\tau), y(\tau))d\tau| < \infty$$

Dissipative systems

The system (5) is said to be dissipative if there exists a so-called storage function $V(x) \geq 0$ such that the following dissipation inequality holds:

$$V(x(t)) \leq V(x(0)) + \int_0^t w(u(\tau), y(\tau))d\tau$$

along all possible trajectories of (5) starting at $x(0)$, for all $x(0)$, $t \geq 0$.

Passive systems

A particular case of dissipative systems are *passive systems*:

Passive systems

Suppose that the system (5) is dissipative with supply rate $w(u, y) = u^T y$ and storage function $V(x(t))$ with $V(0) = 0$; i.e. for all $t \geq 0$ we have that

$$V(x(t)) \leq V(x(0)) + \int_0^t u(\tau)^\top y(\tau) d\tau,$$

Then the system is passive.

Passive systems are a subclass of dissipative systems with the specific properties

- The supply rate is defined by the product between inputs and outputs $w(u, y) = u^T y$,
- The minimum of the storage function is at the origin $V(0) = 0$.

Passive systems

The dissipation of a passive system may also be explicitly taken into account:

Strictly passive systems

A system (5) is said to be strictly state passive if it is dissipative with supply rate $w = u^\top y$ and the storage function $V(x(t))$ with $V(0) = 0$, and there exists a positive definite function $S(x)$ such that for all $t \geq 0$:

$$V(x(t)) = V(x(0)) + \int_0^t u(\tau)^\top y(\tau)d\tau - \int_0^t S(x(\tau))d\tau,$$

If the equality holds and $S(x) = 0$, then the system is said to be lossless (conservative).

The function $S(x)$ is called the the dissipation rate.

Passive systems

Why are we interested in passive systems?

- Many physical systems are passive with respect to the storage function defined by their physical energy function and with respect to their natural supply rate (given by the physical inputs and outputs),
- Its a non-linear approach (does not require any assumption of linearity),
- The physical energy may be used as a candidate Lyapunov function to analyse stability.
- A “well defined” interconnection of passive system is again a passive system.

Stability of passive systems

Recall the passivity inequality

$$V(x(t)) \leq V(x(0)) + \int_0^t u(\tau)^\top y(\tau) d\tau,$$

if the storage function V is strictly increasing and $x = 0$ is an isolated global minimum, then the output feedback $u = 0$ stabilizes the system at $x = 0$ (LaSalle's inv. Principle).

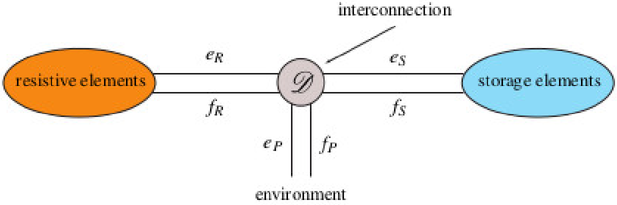
$u = 0$

$$\dot{V} \leq 0$$

For many physical systems: Energy = storage function = Lyapunov function

Port-Hamiltonian control system

Within this formalism a physical system is defined by the interconnection between energy storage elements, resistive elements, and the environment:



This defines a natural space: $\mathbb{F} := \mathbb{F}_S \times \mathbb{F}_R \times \mathbb{F}_P$; $\mathbb{E} := \mathbb{E}_S \times \mathbb{E}_R \times \mathbb{E}_P$

Port-Hamiltonian control systems

The interconnection structure satisfies the power preserving property

$$e_s^\top f_s + e_R^\top f_R + e_p^\top f_p = 0$$

or in terms of the energy storing elements

$$\dot{H}(x(t)) = -e_s^\top f_s = e_R^\top f_R + e_p^\top f_p$$

which yields the energy balance equation

$$H(x(t)) = H(x(0)) + \int_0^t e_R^\top(\tau) f_R(\tau) + e_p^\top(\tau) f_p(\tau) d\tau$$

Dirac structure → port-Hamiltonian systems

The power preserving property is defined by the geometric notion of a Dirac structure, which naturally defines port-Hamiltonian control systems.

Port-Hamiltonian control systems

Explicit PHS

$$\dot{x} = (J - R)\frac{\partial H}{\partial x} + gu(t), \qquad f = (J - R)e + ge_{ext}$$

$$y = g^\top \frac{\partial H}{\partial x} = (g^\top e)$$

Two geometric structures: $J(x)$ and $R(x) = R(x)^\top \geq 0$, the interconnection and damping matrices

The energy-balance is

$$\dot{H} = u^\top(t)y(x(t)) - \frac{\partial H}{\partial x}^\top(x(t))R(x(t))\frac{\partial H}{\partial x}(x(t)),$$

$$\dot{H} \leq u^\top(t)y(t),$$

Dissipative PHS are strictly-passive if the Hamiltonian H is bounded from below.

Stability of strictly passive systems

Recall the strict passivity inequality

$$V(x(t)) \leq V(x(0)) + \int_0^t u(\tau)^\top y(\tau) d\tau - \int_0^t S(x(\tau)) d\tau,$$

if the storage function V is strictly increasing, $x = 0$ is an isolated global minimum and $S(x(t))$ only vanishes at $x = 0$, then the output feedback $u = 0$ asymptomatically stabilizes the system at $x = 0$ (LaSalle's inv. Principle).

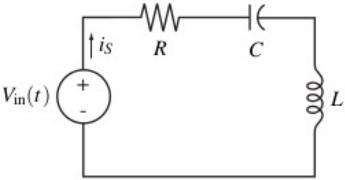
$u = 0$

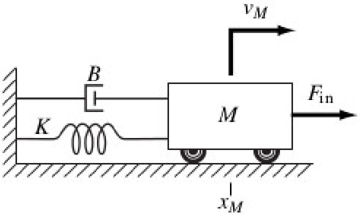
$$\dot{V} \leq -S(x(t))$$

For many physical systems: physical dissipation = dissipation rate

Examples: RLC circuit and MSD system

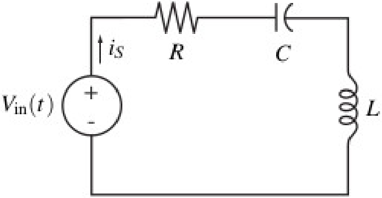
Show that the RLC and the MSD are strictly passive and give the corresponding PHS models.





Example: RLC circuit

Let us consider a simple linear RLC circuit:



Constitutive relations

$$u_s = V_{in}$$
$$u_r = RI_r$$
$$\phi = LI_L$$
$$Q = Cu_C$$

Dynamic relations

$$u_L = \frac{d\phi}{dt}, \quad \text{or in integral form} \quad \phi(t) = \phi(t_0) + \int_0^t u_L(\tau)d\tau$$

$$I_C = \frac{dQ}{dt}, \quad \text{or in integral form} \quad Q(t) = Q(t_0) + \int_0^t I_C(\tau)d\tau$$

Example: RLC circuit

Interconnection relations (Kirchkoff's laws): $\sum u = 0$ voltage law, $\sum i = 0$ current law

Using the interconnection relations together with the constitutive and dynamical relations we obtain the **state space model**

$$\frac{dQ}{dt} = \frac{\phi}{L}$$
$$\frac{d\phi}{dt} = -\frac{Q}{C} - R\frac{\phi}{L} + V_{in}$$

with state variables $x = [Q, \phi]$ and input V_{in}.

- If the initial conditions $Q(t_0)$ and $\phi(t_0)$ are known, together with the profile V_{in}, then the time evolution of the system is fully determined for all $t > t_0$.

Example: RLC circuit

What about the energy of the systems?

Energy = Energy stored in the capacitor + Energy stored in the inductor

$$H(x(t)) = \frac{1}{2}\frac{\phi^2}{L} + \frac{1}{2}\frac{Q^2}{C}$$

The time variation of the energy is given by

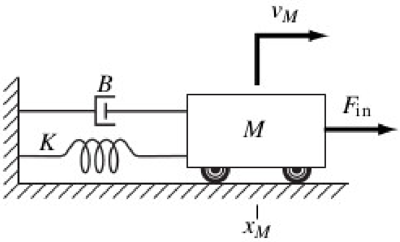
$$\frac{dH(x(t))}{dt} = \frac{\partial H}{\partial x}^\top \frac{dx}{dt} = \left(\frac{Q}{C}\right)\left(\frac{\phi}{L}\right) - \left(\frac{Q}{C}\right)\left(\frac{\phi}{L}\right) + V_{in}\left(\frac{\phi}{L}\right) - R\left(\frac{\phi}{L}\right)^2$$
$$= V_{in}\left(\frac{\phi}{L}\right) - R\left(\frac{\phi}{L}\right)^2 = V_{in}I_L - RI_L^2$$

Hence, the balance equation characterizing the time variation of energy can be written as

$$\boxed{H(t) = H(t_0) + \int_0^t V_{in}(\tau)I_L(\tau)d\tau - \int_0^t RI_L(\tau)^2 d\tau}$$

Example: mass-spring-damper system

Let us consider a simple linear translational MSD system:



Constitutive relations

$$F_s = F_{in}$$
$$F_B = Bv_B$$
$$p = Mv_M$$
$$q = K^{-1}F_K$$

Dynamic relations

$$F_M = \frac{dp}{dt}, \qquad \text{or in integral form} \qquad p(t) = p(t_0) + \int_0^t F_M(\tau)d\tau$$

$$v_K = \frac{dq}{dt}, \qquad \text{or in integral form} \qquad q(t) = q(t_0) + \int_0^t v_K(\tau)d\tau$$

Example: mass-spring-damper system

Using the interconnection relations (Kirchkoff's laws) together with the constitutive and dynamical relations we obtain the **state space model**

$$\frac{dq}{dt} = \frac{p}{M}$$
$$\frac{dp}{dt} = -\frac{q}{K^{-1}} - B\frac{p}{M} + F_{in}$$

with state variables $x = [q, p]$ and input F_{in}.

Example: MSD system

What about the energy of the systems?

Energy = Energy stored in the mass + Energy stored in the spring

$$H(x(t)) = \frac{1}{2}\frac{p}{M}^2 + \frac{1}{2}\frac{q}{K^{-1}}^2$$

The time variation of the energy is given by

$$\begin{aligned} \frac{dH(x(t))}{dt} &= \frac{\partial H}{\partial x}^\top \frac{dx}{dt} = \left(\frac{q}{K^{-1}}\right)\left(\frac{p}{M}\right) - \left(\frac{q}{K^{-1}}\right)\left(\frac{p}{M}\right) + F_{in}\left(\frac{p}{M}\right) - D\left(\frac{p}{M}\right)^2 \\ &= F_{in}\left(\frac{p}{M}\right) - B\left(\frac{p}{M}\right)^2 = F_{in}v_M - Bv_M^2 \end{aligned}$$

The balance equation characterizing the time variation of energy can be written as

$$\boxed{H(t) = H(t_0) + \int_0^t F_{in}(\tau)v_M(\tau)d\tau - \int_0^t Bv_M(\tau)^2 d\tau}$$

Port-modelling of physical systems

Let us look closer to the energy balance

$$\boxed{H(t) = H(t_0) + \underbrace{\int_0^t u_{in}(\tau)y(\tau)d\tau}_{\text{supplied energy}} - \underbrace{\int_0^t R(x)y(\tau)^2 d\tau}_{\text{dissipated energy}}}$$

The balance equations expresses conservation of some physical quantity: Energy, mass, volume, etc...

The existence of balance equations is the base for dissipative and passive system theory.

Example: the RLC circuit

Let us first consider a lossless LC circuit. The energy is

$$H(x(t)) = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}\frac{\phi^2}{L}$$

The interconnection structure just characterize the exchange of energy between the inductor and the capacitor:

$$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

The internal dynamics of the system is then given by

$$\dot{x} = J\frac{\partial H}{\partial x} = J\begin{bmatrix} \frac{Q}{C} \\ \frac{\phi}{L} \end{bmatrix} = \begin{bmatrix} \frac{\phi}{L} \\ -\frac{Q}{C} \end{bmatrix}$$

Example: the RLC circuit

Let us consider the complete RLC circuit, with dissipation and input port. The energy remains the same

$$H(x(t)) = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}\frac{\phi^2}{L}$$

The interconnection structure just characterize the exchange of energy between the inductor and the capacitor, but in this case we have to add an additional structure matrix that characterizes the dissipation of the system and an input vector field

$$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \qquad R = \begin{bmatrix} 0 & 0 \\ 0 & R \end{bmatrix}, \qquad gu = \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

The complete dynamics of the system is now given by

$$\dot{x} = (J - R)\frac{\partial H}{\partial x} + gu = (J - R)\begin{bmatrix} \frac{Q}{C} \\ \frac{\phi}{L} \end{bmatrix} + gu = \begin{bmatrix} \frac{\phi}{L} \\ -\frac{Q}{C} - R\frac{\phi}{L} + V_{in} \end{bmatrix}$$

Example: the MSD system

Let us first consider a lossless MS system. The energy is

$$H(x(t)) = \frac{1}{2}\frac{q}{K^{-1}}^{2} + \frac{1}{2}\frac{p}{M}^{2}$$

The interconnection structure just characterize the exchange of energy between the mass and the spring:

$$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

The internal dynamics of the system is then given by

$$\dot{x} = J\frac{\partial H}{\partial x} = J\begin{bmatrix} \frac{q}{K^{-1}} \\ \frac{p}{M} \end{bmatrix} = \begin{bmatrix} \frac{p}{M} \\ -\frac{q}{K^{-1}} \end{bmatrix}$$

Example: the MSD system

Let us consider the complete MSD system, with dissipation and input port. The energy remains the same

$$H(x(t)) = \frac{1}{2}\frac{q}{K^{-1}}^2 + \frac{1}{2}\frac{p}{M}^2$$

The interconnection structure remains the same, but in this case we have to add an additional structure matrix that characterizes the dissipation of the system and an input vector field

$$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \qquad R = \begin{bmatrix} 0 & 0 \\ 0 & B \end{bmatrix}, \qquad gu = \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

The complete dynamics of the system is now given by

$$\dot{x} = (J - R)\frac{\partial H}{\partial x} + gu = (J - R)\begin{bmatrix} \frac{q}{K^{-1}} \\ \frac{p}{M} \end{bmatrix} + gu = \begin{bmatrix} \frac{p}{M} \\ -\frac{q}{K^{-1}} - B\frac{p}{M} + F_{in} \end{bmatrix}$$

Examples: RLC circuit and MSD system

$$H(x(t)) = \frac{1}{2}\frac{\phi^2}{L} + \frac{1}{2}\frac{Q^2}{C} \geq 0, \qquad H(0) = 0.$$

Hence H qualifies as a potential storage function. Now,

$$\boxed{H(t) = H(t_0) + \int_0^t V_{in}(\tau) I_L(\tau) d\tau - \int_0^t R I_L(\tau)^2 d\tau.}$$

The system is passive if we choose $u = V_{in}$ and $y = I_L$:

$$H(t) \leq H(t_0) + \int_0^t V_{in}(\tau) I_L(\tau) d\tau.$$

Furthermore, if the we choose the dissipation rate as $S(x) = R I_L(\tau)^2$, then the system is strictly passive

$$H(t) = H(t_0) + \underbrace{\int_0^t u(\tau) y(\tau) d\tau}_{\text{supplied energy}} - \underbrace{\int_0^t S(x(\tau)) d\tau}_{\text{dissipated energy}} .$$

Examples: RLC circuit and MSD system

$$H(x(t)) = \frac{1}{2}\frac{q}{K^{-1}}^{2} + \frac{1}{2}\frac{p}{M}^{2} \geq 0, \qquad H(0) = 0.$$

Hence H qualifies as a potential storage function. Now,

$$\boxed{H(t) = H(t_0) + \int_0^t F_{in}(\tau)v_M(\tau)d\tau - \int_0^t Bv_M(\tau)^2 d\tau.}$$

The system is passive if we choose $u = F_{in}$ and $y = v_M$:

$$H(t) \leq H(t_0) + \int_0^t F_{in}(\tau)v_M(\tau)d\tau.$$

Furthermore, if the we choose the dissipation rate as $S(x) = Bv_M(\tau)^2$, then the system is strictly passive

$$H(t) = H(t_0) + \underbrace{\int_0^t u(\tau)y(\tau)d\tau}_{\text{supplied energy}} - \underbrace{\int_0^t S(x(\tau))d\tau}_{\text{dissipated energy}}.$$

Examples: RLC circuit and MSD system

Some remarks

- The chosen inputs and outputs correspond to the physical input and outputs of the system: input voltage and input force / current in the inductor and velocity of the mass
- If we eliminate the resistive components, resistor (R) and damper (B), the supply rate is zero and the system is a lossless (conservative) passive system. Indeed,

$$H(t) = H(t_0) + \underbrace{\int_0^t u(\tau)y(\tau)d\tau}_{\text{supplied energy}}$$

  i.e., the energy is conserved.
- The product $u^\top y$ has the units of **power**, i.e., it defines a power product. This has strong implications for modelling: if the input and outputs define power products the power preserving interconnection of physical (passive) systems defines again a physical (passive) system.

Stabilization of passive systems

- What if we want to increase the rate of convergence?: damping injection,
- What if we want to stabilize at some different equilibrium point, $x = x^*$, $x^* \neq 0$: Energy shaping,
- What if $S(x)$ vanishes for some $x \neq 0$ or $S(x) = 0$?: damping injection + Energy shaping

Stabilization of PHS: Damping injection

Consider the energy balance equation of a passive system:

$$H(t) = H(t_0) + \underbrace{\int_0^t u(\tau)y(\tau)d\tau}_{\text{supplied energy}} - \underbrace{\int_0^t S(x(\tau))d\tau}_{\text{dissipated energy}} .$$

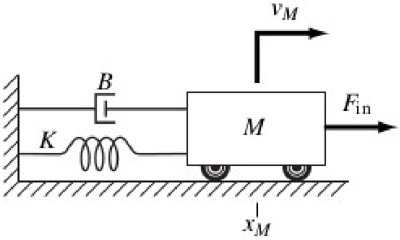
And assume that $H(x)$ qualifies as a Lyapunov function candidate. If we select the input $u = -Ky$, with K a positive definite constant matrix, then the energy balance equation becomes:

$$H(t) = H(t_0) \underbrace{-K\int_0^t y^2(\tau)d\tau}_{\text{controller}} - \underbrace{\int_0^t S(x(\tau))d\tau}_{\text{dissipated energy}} ,$$

$$H(t) = H(t_0) - \underbrace{\int_0^t \left(Ky^2(\tau)d\tau + S(x(\tau))\right) d\tau}_{\text{dissipated energy}}.$$

Example: mass-spring-damper system

Let us consider a simple linear translational MSD system:



Constitutive relations

$$F_s = F_{in}$$
$$F_B = Bv_B$$
$$p = Mv_M$$
$$q = K^{-1}F_K$$

Dynamic relations

$$F_M = \frac{dp}{dt}, \quad \text{or in integral form} \quad p(t) = p(t_0) + \int_0^t F_M(\tau)d\tau$$

$$v_K = \frac{dq}{dt}, \quad \text{or in integral form} \quad q(t) = q(t_0) + \int_0^t v_K(\tau)d\tau$$

Examples: the MSD system

$$H(x(t)) = \frac{1}{2}\frac{q}{K^{-1}}^{2} + \frac{1}{2}\frac{p}{M}^{2} \geq 0, \qquad H(0) = 0.$$

Hence H qualifies as a storage function and as a candidate Lyapunov function. Now,

$$\boxed{H(t) = H(t_0) + \int_0^t F_{in}(\tau)v_M(\tau)d\tau - \int_0^t Bv_M(\tau)^2 d\tau.}$$

The system is passive if we choose $u = F_{in}$ and $y = v_M$, and furthermore, if we select $u = -Ky$, $(F_{in} = -Kv_M)$, then

$$\begin{aligned} H(t) &= H(t_0) - \int_0^t \left(Kv_M^2(\tau) + Bv_M^2(\tau)\right) d\tau. \\ &= H(t_0) - \int_0^t \underbrace{(K+B)}_{B'} v_M^2(\tau)d\tau \end{aligned}$$

We have changed (increased) the system's natural damping.

Stabilization of PHS: Energy Shaping

Consider the energy balance equation of a passive system:

$$\underbrace{H(t) - H(t_0)}_{\text{stored energy}} = \underbrace{\int_0^t u(\tau)y(\tau)d\tau}_{\text{supplied energy}} - \underbrace{\int_0^t S(x(\tau))d\tau}_{\text{dissipated energy}} .$$

Assume that we want to change the closed-loop equilibrium to some forced (controlled) equilibrium $x = x^*$. In that case $H(x^*) \neq 0$, hence $H(x)$ can no longer be used as Lyapunov function!

$\boxed{\text{We need to consider a new Lyapunov function candidate}}$

Stabilization of PHS: Energy shaping

Let us consider the energy balance equation and assume we have no dissipation

$$H(t) - H(t_0) = \underbrace{\int_0^t u(\tau)y(\tau)d\tau}_{\text{controller}}$$

The idea is to construct a new energy function, by using the (state) feedback $u = \beta(x)$

$$H_d(x, x^*) = H(x) - \int_0^t \beta(x(\tau))y(\tau)d\tau$$

such that H_d, with $H_d(x^*) = 0$ qualifies as a Lyapunov function for the closed-loop system.

Stabilization of PHS: Energy shaping

If this function exist (...why should it exist?) it will be a state function such that $H_d(x) = H(x) + H_a(x)$, hence

$$H_a(x, x^*) = -\int_0^t \beta(x(\tau))y(\tau)d\tau$$

The time derivative of $H_d(x)$ along the trajectories of the system is given by

$$\dot{H}_d = \dot{H} + \dot{H}_a = \dot{H} + \frac{\partial H_a}{\partial x}^\top \dot{x}$$

$$\Rightarrow \frac{\partial H_a}{\partial x}^\top \dot{x} = -\beta(x)y$$

Hence, for dynamical systems of the form $\dot{x} = f(x, u)$, $y = h(x)$, in order for the function H_a to exist, the following PDE should be satisfied

$$\boxed{\frac{\partial H_a}{\partial x}^\top f(x, \beta(x)) = -\beta(x)h(x)}$$

Stabilization of PHS: Energy shaping

Some remarks

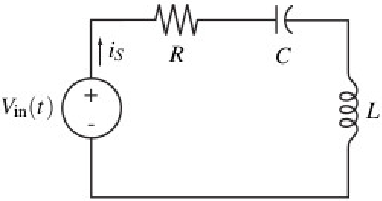
- Energy shaping requires the solution of a PDE: the matching equation. Not an easy task for general non-linear systems

$$H_a(x, x^*) = -\int_0^t \beta(x(\tau))y(\tau)d\tau$$

- The existence of solutions for the PDE is strongly related with the existence of physical invariants. In the case of port-Hamiltonian systems: Casimir functions.
- For systems arising from physical applications the energy shaping technique has been proven to be a powerful stabilization method.

Example: RLC circuit

Let us consider a simple linear RLC circuit:



Constitutive relations

$$
\begin{aligned}
u_s &= V_{in} \\
u_r &= RI_r \\
\phi &= LI_L \\
Q &= Cu_C
\end{aligned}
$$

Dynamic relations

$$u_L = \frac{d\phi}{dt}, \quad \text{or in integral form} \quad \phi(t) = \phi(t_0) + \int_0^t u_L(\tau)d\tau$$

$$I_C = \frac{dQ}{dt}, \quad \text{or in integral form} \quad Q(t) = Q(t_0) + \int_0^t I_C(\tau)d\tau$$

Example: RLC circuit

The **state space model**

$$\frac{dQ}{dt} = \frac{\phi}{L}$$
$$\frac{d\phi}{dt} = -\frac{Q}{C} - R\frac{\phi}{L} + V_{in}$$

with state variables $x = [Q, \phi]$, output $y = \frac{\phi}{L} = \frac{x_2}{L}$ and input V_{in}. The energy of the system is given by

$$H(x) = \frac{1}{2}\frac{x_1}{C}^2 + \frac{1}{2}\frac{x_2}{L}^2$$

- If $V_{in} = 0$, the natural equilibrium is $x = (0, 0)$. If on other hand $V_{in} = V^*$, the forced equilibrium point is $x = (x_1^*, 0)$, with $x_1^* = CV^*$.

The matching condition $\frac{\partial H_a}{\partial x}^\top f(x,\beta(x)) = -\beta(x)h(x)$

$$\frac{\partial H_a}{\partial x_1}\frac{x_2}{L} - \frac{\partial H_a}{\partial x_2}\left(\frac{x_1}{C} - R\frac{x_2}{L} - \beta(x)\right) = -\frac{x_2}{L}\beta(x)$$

Notice that the forced equilibrium corresponding to the x_2 coordinate already is a minimum of the physical energy $H(x)$, hence we only need to shape the closed-loop energy in the x_1 coordinate. Hence

$$\boxed{H_a = H_a(x_1)}$$

and the matching equation becomes

$$\frac{\partial H_a}{\partial x_1}\frac{x_2}{L} = -\frac{x_2}{L}\beta(x)$$

Hence, the function H_a exists if the feedback is chosen as $\boxed{\beta(x) = -\frac{\partial H_a}{\partial x_1}}$

Example: RLC circuit

Beautiful!

The matching equation (PDE) is automatically solved for any $H_a = H_a(x_1)$ provided that the state feedback is of the form $\beta(x) = -\frac{\partial H_a}{\partial x_1}$.

- It only remains to select $H_a(x_1)$ such that $H_d = H + H_a$ has a minimum at $x^* = (x_1^*, 0)$.

Recall that the open-loop energy function is

$$H(x) = \frac{1}{2}\frac{x_1}{C}^2 + \frac{1}{2}\frac{x_2}{L}^2$$

Hence if we chose

$$H_a(x_1) = \frac{1}{2C_a}x_1^2 - \left(\frac{1}{C_a} + \frac{1}{C}\right)x_1 x_1^*$$

The closed-loop energy function $H_d = H + H_a$

$$\boxed{H_d(x) = \frac{1}{2}\frac{(x_1 - x_1^*)}{(C + C_a)}^2 + \frac{1}{2}\frac{x_2}{L}^2}$$

Example: RLC circuit

$H_d(x, x^*)$ has a minimum at $x^* = (x_1^*, 0)$ if and only if $C_a > -C$

The resulting controller is

$$\boxed{\beta(x) = -\frac{\partial H_a}{\partial x_1}(x) = -\frac{1}{2C_a}x_1^2 - \left(\frac{1}{C_a} + \frac{1}{C}\right) x_1 x_1^*}$$

Final remarks

- We have revised some concepts from passivity based control techniques: Damping injection and Energy Shaping
- We have exploited the natural passivity of the system to design stabilizing controllers
- Works well in many applications, but.... we did not see the use of physical invariants, the dissipation obstacle,...

What remains for tomorrow

- Control by interconnection
- IDA-PBC
- Irreversible port-Hamiltonian systems...

1. Stability: definitions

2. Stabilization of PHS

3. Control by interconnection

4. Interconnection and Damping Assignment – Passivity Based Control

5. Final remarks and future work

Port-Hamiltonian control systems

Let us recall the state space model of a port-Hamiltonian control system

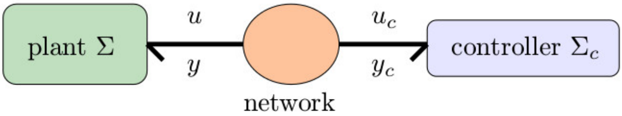
$$\boxed{\begin{aligned} \dot{x} &= \left(J(x) - R(x)\right)\frac{\partial H}{\partial x}(x) + g(x)u, \\ y &= g^\top(x)\frac{\partial H}{\partial x}(x), \end{aligned}}$$

where where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{u} \in \mathbb{R}^m$, $m < n$, is the control action, $H : \mathbb{R}^n \to \mathbb{R}$ is the total stored energy, $J(x) = -J(x)^\top$ is the $n \times n$ natural interconnection matrix, $R(x) = R(x)^\top \geq 0$ is the $n \times n$ damping matrix, $g(x)$, is the $n \times m$ input map and $\mathbf{u}, \mathbf{y} \in \mathbb{R}^m$, are conjugated variables whose product has units of power.

$$\boxed{\begin{aligned} \dot{H} &= u^\top y - \frac{\partial H}{\partial x}^\top R \frac{\partial H}{\partial x}, \\ \dot{H} &\leq u^\top y, \end{aligned}}$$

Control by interconnection

A controlled system may be viewed as a plant system interconnected with a control system exchanging energy

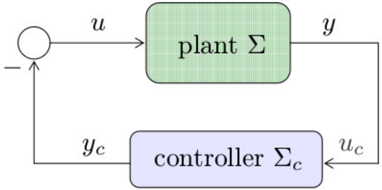


The interconnection is power continuous if

$$\boxed{u^\top(t)y(t) + u_c^\top(t)y_c(t) = 0, \qquad \forall t}$$

Control by interconnection

For instance: a negative feedback

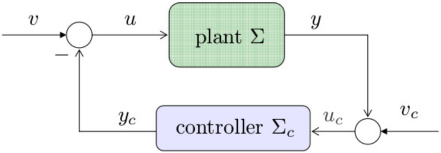


The negative feedback defines the following relation

$$\begin{array}{l} u = -y_c \\ y = u_c \end{array} \Rightarrow u^\top y + u_c^\top y_c = -y_c^\top y + y^\top y_c = 0$$

Control by interconnection

Let us consider the feedback



Now $u = v - y_c$ and $u_c = y + v_c$. Let the plant and controller have state variables x and ξ and energy functions $H(x)$ and $H(\xi)$. If the maps $u \to y$ and $u_c \to y_c$ are passive,

Then the map $(v, v_c) \to (y, y_c)$ is passive with energy function $H_d(x, \xi) = H(x) + H(\xi)$.

Control by interconnection

Assume that the plant and the controller are PHS

$$\Sigma : \quad \begin{cases} \dot{x} = [J(x) - R(x)] \dfrac{\partial H}{\partial x}(x) + g(x)u \\ y = g^\top(x) \dfrac{\partial H}{\partial x}(x) \end{cases} \quad x \in \mathcal{X}$$

$$\Sigma_c : \quad \begin{cases} \dot{\xi} = [J_c(\xi) - R_c(\xi)] \dfrac{\partial H_c}{\partial \xi}(\xi) + g_c(\xi)u_c \\ y_c = g_c^\top(\xi) \dfrac{\partial H_c}{\partial \xi}(\xi) \end{cases} \quad \xi \in \mathcal{X}_c$$

Booth are passive systems, so a power preserving interconnection, $u = -y_c$, $y = u_c$, yields a passive closed-loop system.

Control by interconnection

The closed-loop systems looks

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = \left(\underbrace{\begin{bmatrix} J(x) & -g(x)g_c^\top(\xi) \\ g_c(\xi)g^\top(x) & J_c(\xi) \end{bmatrix}}_{J_{cl}(x,\xi)} - \underbrace{\begin{bmatrix} R(x) & 0 \\ 0 & R_c(\xi) \end{bmatrix}}_{R_{cl}(x,\xi)} \right) \begin{bmatrix} \frac{\partial H_d}{\partial x}(x) \\ \frac{\partial H_d}{\partial \xi}(\xi) \end{bmatrix}$$

$$\begin{bmatrix} y \\ y_c \end{bmatrix} = \underbrace{\begin{bmatrix} g(x) & 0 \\ 0 & g_c(\xi) \end{bmatrix}}_{g_{cl}} \begin{bmatrix} \frac{\partial H_d}{\partial x}(x) \\ \frac{\partial H_d}{\partial \xi}(\xi) \end{bmatrix}$$

With total energy function

$$\boxed{H_d(x,\xi) = H(x) + H_c(\xi)}$$

We may equivalently write the closed-loop system as

$$\dot{w} = (J_{cl} - R_{cl}) \frac{\partial H_d}{\partial w}, \qquad y_{cl} = g_{cl}^\top \frac{\partial H_d}{\partial w}$$

with $w = [x\ \xi]$.

Control by interconnection

So, what now?

We would like to get an energy function in terms of x only: $H_d = H_d(x)$, so that we can assign the minimum at a desired point and characterize it in terms of the plant system.

In order achieve this, we must restrict the dynamics to a submanifold of the (x, ξ) space parametrized by x. This means that we are looking for a submanifold

$$\boxed{\Omega_C = (x, \xi) : \xi = F(x) - C}$$

which is dynamically invariant, i.e.,

$$\left(\frac{\partial F_i}{\partial x}^\top \dot{x} - \dot{\xi}_i \right)_{\xi = F_i(x) - C} = 0$$

Control by interconnection

Casimir functions

Let us look for structural invariants that relates each state of the controller with the states of the plant:

$$C_i(x,\xi_i) = F_i(x) - \xi_i$$

In order to relate all the states of the controller with the state of the plant we define $F(x) = [F_1(x), F_2(x), \ldots, F_{n_c}(x)]$, and define the following Casimir function

$$C = \sum_{i=1}^{n}(F_i(x) - \xi_i) = \sum_{i=1}^{n} C_i(x,\xi_i)$$

C is an invariant of the system, hence

$$\dot{C} = \frac{\partial C}{\partial w}^{\top}\dot{w} = \frac{\partial C}{\partial w}^{\top}\left(J_{cl}\frac{\partial H_{cl}}{\partial w}\right) = 0$$

But furthermore, C is a structural invariants, so it should be invariant with respect to the structure of the system:

$$\frac{\partial C}{\partial w}^{\top} J_{cl} = 0$$

Control by interconnection

Casimir functions

Let us look for structural invariants that relates each state of the controller with the states of the plant:

$$C = \sum_{i=1}^{n} C_i(x, \xi_i) = \sum_{i=1}^{n} (F_i(x) - \xi_i)$$

we obtain the following matching condition

$$\underbrace{\begin{bmatrix} \frac{\partial F}{\partial x}^\top(x) & -\mathbb{I} \end{bmatrix} \begin{bmatrix} J(x) - R(x) & -g(x)g_C^\top(\xi) \\ g_C(\xi)g^\top(x) & J_C(\xi) - R_C(\xi) \end{bmatrix}}_{\text{Matching condition}} \begin{bmatrix} \frac{\partial H_d}{\partial x}(x) \\ \frac{\partial H_d}{\partial \xi}(\xi) \end{bmatrix} = 0$$

- Only the term in blue is considered in the matching condition because we want the Casimir functions to be structural invariants of the system: not depend on $H_d(x, \xi)$.

Control by interconnection

The condition for existence of Casimir functions for the closed loop system

$$\begin{bmatrix} \frac{\partial F}{\partial x}^\top(x) & -\mathbb{I} \end{bmatrix} \begin{bmatrix} J(x) - R(x) & -g(x)g_C^\top(\xi) \\ g_C(\xi)g^\top(x) & J_C(\xi) - R_C(\xi) \end{bmatrix} = 0$$

may be written out as

Matching equations

$$\frac{\partial F}{\partial x}^\top(x)J(x)\frac{\partial F}{\partial x}(x) = J_c(\xi)$$

$$R(x)\frac{\partial F}{\partial x}(x) = 0 \qquad \text{Dissipation obstacle!}$$

$$R_c(\xi) = 0$$

$$\frac{\partial F}{\partial x}^\top(x)J(x) = g_c(\xi)g^\top(x)$$

Control by interconnection

The closed-loop dynamic then takes the form

$$\dot{x} = \Big[J(x) - R(x)\Big]\frac{\partial H}{\partial x}(x) - g(x)g_C^\top(\xi)\frac{\partial H_c}{\partial \xi}(\xi)$$

Using the second and fourth M.C. we get

$$\dot{x} = \Big[J(x) - R(x)\Big]\left(\frac{\partial H}{\partial x}(x) + \frac{\partial F}{\partial x}(x)\frac{\partial H_c}{\partial \xi}(\xi)\right)$$

Since $\xi = F(x) - C$, we use the chain-rule for differentiation to establish

$$\frac{\partial F}{\partial x}(x)\frac{\partial H_c}{\partial \xi}(\xi) = \frac{\partial H_c}{\partial x}(F(x) - C)$$

Control by interconnection

Hence we obtain:

$$\dot{x} = \Big[J(x) - R(x)\Big] \left(\frac{\partial H}{\partial x}(x) + \frac{\partial H_c}{\partial x}(F(x) - C) \right)$$

Or equivalently

$$\boxed{\dot{x} = \Big[J(x) - R(x)\Big] \frac{\partial H_d}{\partial x}(x)}$$

With closed-loop energy $H_d(x) = H(x) + H_c(F(x) - C)$.

Control by interconnection

Let us interpret the control in terms of ***energy balancing***. Since $R_c = 0$, the energy balance equation of the controller is

$$\frac{dH_c}{dt} = u_c^\top y_c$$

Hence, along any invariant submanifold Ω_C, we have

$$\frac{dH_d}{dt} = \frac{dH}{dt} + \frac{dH_c}{dt} = \frac{dH}{dt} - u^\top y \qquad (u_c^\top y_c = -u^\top y)$$

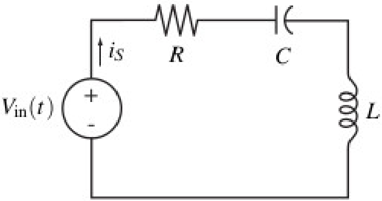
and integrating (up to some constant) we obtain

$$\boxed{H_d(t) = H(t) \underbrace{- \int_0^t u^\top(\tau) y(\tau) d\tau}_{H_c}}$$

We obtain the general M.E!: $H_c(t) = -\int_0^t u^\top(\tau) y(\tau) d\tau$.

Example: RLC circuit

Let us consider a simple linear RLC circuit:



Constitutive relations

$$u_s = V_{in}$$
$$u_r = RI_r$$
$$\phi = LI_L$$
$$Q = Cu_C$$

Dynamic relations

$$u_L = \frac{d\phi}{dt}, \quad \text{or in integral form} \quad \phi(t) = \phi(t_0) + \int_0^t u_L(\tau)d\tau$$

$$I_C = \frac{dQ}{dt}, \quad \text{or in integral form} \quad Q(t) = Q(t_0) + \int_0^t I_C(\tau)d\tau$$

Example: the RLC circuit

Let us consider a RLC circuit, with dissipation and input port. The energy is

$$H(x(t)) = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}\frac{\phi^2}{L}$$

The interconnection and dissipation matrix and input vector field are

$$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \qquad R = \begin{bmatrix} 0 & 0 \\ 0 & R \end{bmatrix}, \qquad gu = \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

and the dynamic is

$$\dot{x} = (J - R)\frac{\partial H}{\partial x} + gu = (J - R)\begin{bmatrix} \frac{Q}{C} \\ \frac{\phi}{L} \end{bmatrix} + gu = \begin{bmatrix} \frac{\phi}{L} \\ -\frac{Q}{C} - R\frac{\phi}{L} + V_{in} \end{bmatrix}$$

with state variables $x = [Q, \phi]$, output $y = \frac{\phi}{L} = \frac{x_2}{L}$ and input $u = V_{in}$

- If $V_{in} = 0$, the natural equilibrium is $x = (0, 0)$. If on other hand $V_{in} = V^*$, the forced equilibrium point is $x = (x_1^*, 0)$, with $x_1^* = CV^*$.

Example: RLC circuit

Let us synthesis the controller using the M.C.s.

- From physical considerations we know that we only need to shape the x_1 coordinate: F is only one scalar function.
- From M.E.2 we obtain that $\frac{\partial F}{\partial x_2} = 0$.
- Then, from M.E.1. we obtain that $J_c = 0$, and from M.E.3 that $R_c = 0$.
- Finally from M.E.4 we have that $\frac{\partial F}{\partial x_1} = g_c(\xi)$ and that $\xi \in \mathbb{R}$.

We would like to have

$$H_c(x_1) = \frac{1}{2C_a}x_1^2 - \left(\frac{1}{C_a} + \frac{1}{C}\right) x_1 x_1^*, \quad \text{such that} \quad \boxed{H_d(x) = \frac{1}{2}\frac{(x_1 - x_1^*)^2}{(C + C_a)} + \frac{1}{2}\frac{x_2}{L}^2}$$

This is achieved if we select $F(x_1) = x_1$ and $C = 0$ on the invariant submanifold such that $\xi = x_1$. The control system is then given by (using condition $\frac{\partial F}{\partial x_1} = g_c(\xi)$)

$$\dot{\xi} = u_c$$
$$y_c = \frac{\partial H_c}{\partial \xi}$$

The pendulum without damping

The dynamic equations

$$\dot{q} = \frac{p}{m}$$
$$\dot{p} = -mg\sin(q) + u$$

with state variables $x = [p, q]$, with q the configuration and p the momentum.

Propose a PH model and design a stabilizing controller using the Casimir method and a control system given by:

$$\dot{\xi} = u_c$$
$$y_c = \frac{\partial H_c}{\partial \xi}$$

Example: pendulum (model)

Consider the pendulum without damping

$$\begin{aligned}\dot{q} &= \frac{p}{m}\\ \dot{p} &= -mg\sin q + u\end{aligned}$$

with state variables $x = [q, p]^T$ with q the configuration and p the momentum.
The port Hamiltonian model is:

$$\begin{aligned}\frac{d}{dt}\begin{pmatrix} q \\ p \end{pmatrix} &= \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{J}\begin{pmatrix} \frac{\partial H_0}{\partial q} \\ \frac{\partial H_0}{\partial p} \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{g} u\\ y &= (0\,1)\begin{pmatrix} \frac{\partial H_0}{\partial q} \\ \frac{\partial H_0}{\partial p} \end{pmatrix} = \frac{p}{m}\end{aligned}$$

with Hamiltonian :$H_0(q,\, p) = mg(1 - \cos q) + \frac{1}{2m}p^2$

pendulum (extended Casimir functions)

Recall: we look for Casimir functions such as:

$$C\left(q,\, p,\, x_c\right) = F\left(q,\, p\right) - x_c$$

that is functions $F\left(q,\, p\right)$ satisfying:

1. input vector field is Hamiltonian: $g = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = J\left(x\right) \frac{\partial F}{\partial x} = \begin{pmatrix} \frac{\partial F}{\partial p} \\ -\frac{\partial F}{\partial q} \end{pmatrix}$
2. the gradient of F is transversal to g: $L_g F\left(x\right) = 0 = \frac{\partial F}{\partial x}^t g\left(x\right) = \frac{\partial F}{\partial p}$

hence a generating function is: $F(q,\, p) = -q$

pendulum (control)

For the controller design we choose a function $H_C(x_c)$ such that $H_d\,(x) = H_0\,(x) + H_c \circ F$ has a minimum at the desired equilibrium $x_* = (x_1^*, 0)$. The simplest choice is given by

$$H_C(x_c) = mg(\cos x_c - 1) + \frac{1}{2}(x_c + x_1^*)^2$$

The control is finally obtained is:

$$u = -\frac{\partial H_C}{\partial x_c}(x_c)\,|_{x_c=-q} = mg\sin q - (q - x_1^*)$$

which is the well-known “proportional plus gravity compensation control

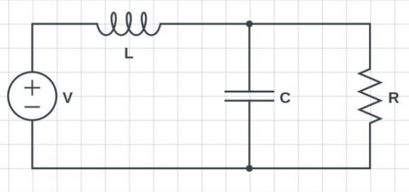
Control by interconnection

Remarks

- The port-Hamiltonian structure provide important information for finding the solutions of the control system,
- The control has physical interpretation in terms of interconnection and energy balancing
- The Casimir method can be used to analyse new stability profiles of interconnected systems

Interconnection and Damping Assignment – Passivity Based Control

Consider the following parallel RLC circuit



with $x = [q_C, \phi_L]$, the charge in the capacitor and the flux in the inductance

The PH model is

$$\begin{bmatrix} \dot{q}_C \\ \dot{\phi}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{R} & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q_C} \\ \frac{\partial H}{\partial \phi_L} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q_C} \\ \frac{\partial H}{\partial \phi_L} \end{bmatrix}$$

with

$$H(q_C, \phi_L) = \frac{q_C^2}{2C} + \frac{\phi_L^2}{2L},$$

the total electromagnetic energy.

Interconnection and Damping Assignment – Passivity Based Control

Recall the dissipation obstacle

$$R(x)\frac{\partial F}{\partial x}(x) = 0.$$

For the parallel RLCS circuit it becomes:

$$\frac{\partial F}{\partial q_C}(q_C, \phi_L) = 0 \quad \Rightarrow F = F(\phi_L).$$

It's only possible to shape the energy in the direction of one coordinate (ϕ_L). This is known as the dissipation obstacle.

Physical interpretation: admissible equilibria are of the form $q_C^* = CV^*$, $\phi_L = \frac{L}{R}V^*$ for any constant V^*. The power delivered by the source, $V^*\frac{\phi_L}{L}$, is nonzero at any equilibrium except for the trivial one. Hence, the source has to provide an infinite amount of energy to keep any nontrivial equilibrium point (Impossible). Notice that pure mechanical systems are free of this problem: any equilibrium point has velocities equal to zero.

Interconnection and Damping Assignment – Passivity Based Control

Overcome the dissipation obstacle using a

State modulated control by interconnection (state modulated source)

IDA-PBC objective

Find a static state-feedback control $u(x) = \beta(x)$ such that the closed-loop dynamics is a PH system with interconnection and dissipation of the form

$$\dot{x} = (J_d - R_d)\frac{\partial H_d}{\partial x},$$

$H_d(x)$, has a strict local minimum at x^*,
$J_d(x, u) = -J_d(x, u)^T$, the *desired* interconnection matrix,
$R_d(x, u) = R_d(x, u)^T \geq 0$, the *desired* dissipation matrix,

Interconnection and Damping Assignment – Passivity Based Control

The procedure consists in the matching of the open and (desired) closed-loop vector fields:

$$(J(x) - R(x))\frac{\partial H}{\partial x}(x) + g(x)\beta(x) = (J_d(x) - R_d(x))\frac{\partial H_d}{\partial x}(x)$$

with $u = \beta(x)$ the state modulated source. Define

$$\begin{aligned} J_a &= J_d - J, & J_a &= -J_a^\top \\ R_a &= R_d - R, & R_a &= R_a^\top \geq 0 \\ H_a &= H_d - H & & \end{aligned}$$

Then the matching condition becomes

$$\boxed{(J_a(x) - R_a(x))\frac{\partial H}{\partial x}(x) + g(x)\beta(x) = \Big((J(x) + J_a(x)) - (R(x) + R_a(x))\Big)\frac{\partial H_a}{\partial x}(x)}$$

with design parameters J_a, R_a and H_a.

IDA-PBC: Proposition

Suppose that the following PDE is verified

$$g^{\perp}(x)\left(J_a(x) - R_a(x)\right)\frac{\partial H}{\partial x}(x) = g^{\perp}(x)\Big(\left(J(x) + J_a(x)\right) - \left(R(x) + R_a(x)\right)\Big)\frac{\partial H_a}{\partial x}(x)$$

where $g^{\perp}(x)g(x) = 0$ and $H_d(x)$ is such that

$$\frac{\partial H_d}{\partial x}(x^*) = 0, \qquad \frac{\partial^2 H_d}{\partial x^2}(x^*) > 0,$$

then the control

$$\boxed{u = \beta(x) = (g^{\top} g)^{-1} g^{\top}\left[(J_d - R_d)\frac{\partial H_d}{\partial x} - (J - R)\frac{\partial H}{\partial x}\right]}$$

is such that the closed-loop system takes the PH form

$$\dot{x} = (J_d - R_d)\frac{\partial H_d}{\partial x},$$

And closed-loop energy

$$\boxed{\dot{H}_d = -\frac{\partial H_d^{\top}}{\partial x} R_d \frac{\partial H_d}{\partial x} < 0, \ \forall x \neq x^* \quad \text{and} \quad \dot{H}_d(x^*) = 0.}$$

Interconnection and Damping Assignment – Passivity Based Control

Notice that for systems linear in the control may be interpreted as the optimal solution to the linear least square problem

$$g(x)\beta(x) = \underbrace{\left[\left(J_d(x) - R_d(x)\right) \frac{\partial H_d}{\partial x}(x) - \left(J(x) - R(x)\right) \frac{\partial H}{\partial x}(x) \right]}_{b}$$

with

$$\beta(x) = g^\dagger b, \qquad \text{subject to the ME} \quad (I - g^\dagger g) b = 0$$

where

$$g^\dagger = (g^\top g)^{-1} g^\top : \quad \text{Moore-Penrose pseudo-inverse of } g \text{ and } g^\dagger g^\top \text{ projector into } rang(g)$$

$$(I - g^\dagger g) : \quad \text{family of anihilators of } g \text{ and projector into } ker(g)$$

Remarks

Degrees of freedom in the design

- J_d and R_d are free–up to the constraint of skew–symmetry and positive semidefiniteness, respectively.
- H_d may be totally, or partially fixed, provided we can ensure $\frac{\partial H_d}{\partial x}(x^*) = 0$, $\frac{\partial^2 H_d}{\partial x^2}(x^*) \geq 0$ and probably a properness condition.
- there is an additional degree of freedom in $g^{\perp}(x)$ which is not uniquely defined by $g(x)$.

$\boxed{\text{Attention: Requires the solution of a quasilinear PDE}}$

the method of characteristics...

Interconnection and Damping Assignment – Passivity Based Control

Notice that the method can be equally applied to general input affine non-linear systems

$$f(x) + g(x)\beta(x) = \left(J_d(x) - R_d(x)\right)\frac{\partial H_d}{\partial x}(x)$$

with $f(x)$ a non-linear vector field.

Then the matching condition becomes

$$g^{\perp}(x)f(x) = g^{\perp}(x)\left(J_d(x) - R_d(x)\right)\frac{\partial H_d}{\partial x}(x)$$

and the control

$$\boxed{u = \beta(x) = (g^{\top}g)^{-1}g^{\top}\left[(J_d - R_d)\frac{\partial H_d}{\partial x} - f(x)\right]}$$

Integral Action

IDA–PBC with integral action $u = \beta(x) + v$

$$\dot{v} = -K_I g(x) \frac{\partial H_d}{\partial x}(x)$$

with $K_I = K_I^\top > 0$, preserves stability.

Indeed, define

$$W(x, v) = H_d + \frac{1}{2} v^\top K_I^{-1} v$$

The closed-loop system is given by a power preserving interconnection

$$\boxed{\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} J_d - R_d & gK_I \\ -K_I^\top g^\top & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H_d}{\partial x} \\ \frac{\partial W}{\partial v} \end{bmatrix}}$$

which is again a PHS.

Existing Approaches to Solve the Matching Equation

Non–Parameterized IDA One extreme case

- fix $J_d(x)$, $R_d(x)$ and $g^{\perp}(x)$,
- (ME) becomes a PDE for $H_d(x)$,
- among the family of solutions select one that assigns minimum.

Algebraic IDA At the other extreme:

- fix $H_d(x)$,
- (ME) becomes an algebraic equation in $J_d(x)$, $R_d(x)$ and $g^{\perp}(x)$,

Parameterized IDA Restrict the desired energy function to a certain class,

- for instance, for mechanical systems

$$H_d(q,p) = \frac{1}{2}p^\top M_d^{-1}(q)p + \frac{1}{2}V_d(q),$$

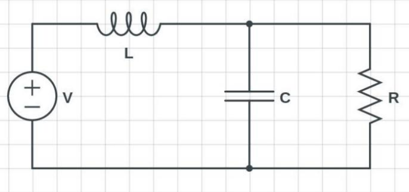
- (ME) becomes a PDE in $M_d(q)$ and $V_d(q)$,
- imposes some constraints on $J_d(x)$ and $R_d(x)$.

Application of **Poincare's Lemma** (applicable for systems NL in u):

$$\nabla H(x) = F_d^{-1}(x) f(x,u).$$

Example: parallel RLCS circuit

Consider the following parallel RLC circuit



with $x = [q_C, \phi_L]$, the charge in the capacitor and the flux in the inductance

The PH model is

$$\begin{bmatrix} \dot{q}_C \\ \dot{\phi}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{R} & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q_C} \\ \frac{\partial H}{\partial \phi_L} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q_C} \\ \frac{\partial H}{\partial \phi_L} \end{bmatrix}$$

with

$$H(q_C, \phi_L) = \frac{q_C^2}{2C} + \frac{\phi_L^2}{2L},$$

the total electromagnetic energy.

Parallel RLCS circuit: matching equation

Choice of structure matrices and associated added Hamiltonian $H_a(x)$:

1. choose added structure matrices

$$J_a(x) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad R_a(x) = \begin{pmatrix} 0 & 0 \\ 0 & R_a \end{pmatrix} \; R_a > -R$$

2. solve a PDE in $H_a(x)$ using the left annihilator $g^{\perp}(x) = \begin{pmatrix} 1 & 0 \end{pmatrix}$ of $g(x) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, the matching equation :

$$-g^{\perp}(x)\,(J_a - R_a)\,\frac{\partial H_0}{\partial x}(x) = g^{\perp}(x)\left[(J(x) + J_a(x)) - (R(x) + R_a(x))\right]\frac{\partial H_a}{\partial x}(x)$$

becomes:

$$0 = \begin{pmatrix} -\frac{1}{R} & 1 \end{pmatrix}\begin{pmatrix} \frac{\partial H_a}{\partial q_c} \\ \frac{\partial H_a}{\partial \phi_L} \end{pmatrix} = -\frac{1}{R}\frac{\partial H_a}{\partial q_c} + \frac{\partial H_a}{\partial \phi_L}$$

Parallel RLCS circuit: closed-loop equilibria

The solutions of the matching equation are:

$$H_a\left(q_c,\ \phi_L\right) = \Phi\left(R\, q_c + \phi_L\right) \qquad \Phi \in C^{\infty}\left(\mathbb{R}\right)$$

hence the closed-loop Hamiltonian is

$$H_d\left(q_C, \phi_L\right) = H\left(q_C, \phi_L\right) + \Phi\left(R\, q_c + \phi_L\right) = \frac{q_C^2}{2C} + \frac{\phi_L^2}{2L} + \Phi\left(R\, q_c + \phi_L\right)$$

The equilibrium in closed-loop is given by:

$$R\,\frac{\phi_L^*}{L} - \frac{q_C^*}{C} = 0 \quad \text{and} \quad \frac{\phi_L^*}{L} + \frac{\partial \Phi}{\partial \xi}\left(\left[1 + \frac{R^2 C}{L}\right]\phi_L^*\right) = 0$$

Parallel RLCS circuit: two possible H

1. Case 1: $\Phi(\xi) = k\,\frac{\xi^2}{2}$ then:

$$\phi^* \in \mathbb{R} \quad \text{and} \quad k = -\frac{(L + R^2 C)}{L^2} < 0$$

   and $\Phi(\xi)$ is concave !

2. Case 2: $\Phi(\xi) = k\,\frac{\xi^4}{4}$ then:

$$\phi^* = \pm\frac{1}{\sqrt{(-k)\,(L + R^2 C)}} \quad \text{and} \quad k = \in \mathbb{R}^*_-$$

   and $\Phi(\xi)$ is again concave !

Parallel RLCS circuit: local positivity of H_d

Consider the Hessian of H_d at the equilibrium:

$$\frac{\partial^2 H_d}{\partial q_c, \phi_L^2} = \begin{pmatrix} \frac{1}{C} + R^2 \frac{\partial^2 \Phi}{\partial \xi^2}\left(R q_c^* + \phi_L^{*2}\right) & R \frac{\partial^2 \Phi}{\partial \xi^2}\left(R q_c^* + \phi_L^{*2}\right) \\ R \frac{\partial^2 \Phi}{\partial \xi^2}\left(R q_c^* + \phi_L^{*2}\right) & \frac{1}{L} + \frac{\partial^2 \Phi}{\partial \xi^2}\left(R q_c^* + \phi_L^{*2}\right) \end{pmatrix}$$

is definite positive iff:

1. either: $\frac{1}{C} + R^2 \frac{\partial^2 \Phi}{\partial \xi^2}\left(R q_c^* + \phi_L^2\right) > 0$ or: $\frac{1}{L} + \frac{\partial^2 \Phi}{\partial \xi^2}\left(R q_c^* + \phi_L^{*2}\right) > 0$
2. and det $\frac{\partial^2 H_d}{\partial q_c, \phi_L^2} > 0$ i.e. : $\frac{1}{LC}\left(1 + \left[L + R^2 C\right] \frac{\partial^2 \Phi}{\partial \xi^2}\left(R q_c^* + \phi_L^{*2}\right)\right) > 0$

which reduces to: $\frac{\partial^2 \Phi}{\partial \xi^2}\left(R q_c^* + \phi_L^2\right) > -\frac{1}{(R^2 C + L)}$

Parallel RLCS circuit: two possible Φ

Check for the two examples the condition : $\frac{\partial^2 \Phi}{\partial \xi^2} \left(Rq_c^* + \phi_L^2 \right) > -\frac{1}{\left(R^2 C + L\right)}$

1. Case 1: $\Phi\left(\xi\right) = k\,\frac{\xi^2}{2}$ the condition reduces to:

$$\left(L + R^2 C\right)^2 < L^2$$

   which is wrong !

2. Case 2: $\Phi\left(\xi\right) = k\,\frac{\xi^4}{4}$ then:

$$(-k) < \frac{1}{\left(L + R^2 C\right)\left(\frac{\phi_L^*}{L}\right)^2}$$

   which leads to a solution !

Parallel RLCS circuit: IDA-PBC control

The control law is given by :

$$\beta(x) = \left[g^t(x)\,g(x)\right]^{-1} g^t(x) \left\{\left[(J(x)+J_a(x)) - (R(x)+R_a(x))\right]\frac{\partial H_a}{\partial x}(x) + (J_a - R_a)\frac{\partial H}{\partial x}(x)\right\}$$

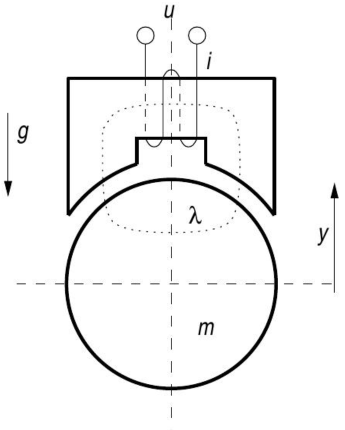
which becomes :

$$\beta(q_C,\phi_L) = 1 \begin{pmatrix} 0 & 1 \end{pmatrix} \left[\begin{pmatrix} -\frac{1}{R} & 1 \\ -1 & -R_a \end{pmatrix}\begin{pmatrix} R\,\frac{\partial\Phi}{\partial\xi}(R\,q_C,+\phi_L) \\ \frac{\partial\Phi}{\partial\xi}(R\,q_C,+\phi_L) \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & R_a \end{pmatrix}\begin{pmatrix} \frac{q_C}{C} \\ \frac{\phi_L}{L} \end{pmatrix}\right]$$

or :

$$\beta(q_C,\phi_L) = (R-R_a)\,\frac{\partial\Phi}{\partial\xi}(R\,q_C,+\phi_L) - R_a\frac{\phi_L}{L}$$

The levitating iron ball in a magnetic field



The mathematical model

$$\dot{\phi}_1 = -Ri + u$$
$$\dot{y} = v$$
$$m\dot{v} = -F_m + mg$$

with u input voltage, R electric resistance, $m$the mass and g the gravitational constant,

$F_m = -\dfrac{\partial W_c}{\partial y}(i, y)$ electro-mechanical coupling

$W_c = \dfrac{1}{2}L(y)i^2$ non-linear inductance

and the non-linear inductance

$$L(y) = L_\infty + \frac{k}{(a+y)},$$

with $L_\infty, a, k > 0$ and $\phi = L(y)i$.

The levitating iron ball: the PH model

The port-Hamiltonian model

set as state variables:
$x_1 = \phi$ total magnetic flux
$x_2 = y$ displacement of the ball
$x_3 = mv = p$ kinetic momentum

$$\frac{dx}{dt} = \left(\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}}_{J} - \underbrace{\begin{bmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{R} \right) \frac{\partial H}{\partial x}(x) + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{g} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \frac{\partial H}{\partial x}(x) = \frac{\partial H}{\partial x_1} = i$$

with total energy: $H(x) = \frac{1}{2\,L(x_2)} {x_1}^2 + \frac{1}{2m} {x_3}^2 - mgx_2$

$$\frac{\partial H_0}{\partial x}(x) = \begin{pmatrix} \frac{x_1}{L(x_2)} \\ \frac{x_3}{m} \\ -\frac{1}{2}\frac{dL}{dx_2}(x_2)\left(\frac{x_1}{L(x_2)}\right)^2 - m\,g \end{pmatrix}$$

current through the coil, i
velocity of the ball
electro-motive+gravity force

The levitating ball: coupling through energy

The coupling between the mechanical and magnetic domain does not occur through the structure matrices

$$J - R = \begin{bmatrix} -R & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \quad \begin{array}{l} \text{magnetic domain} \\ \text{mechanical potential domain} \\ \text{mechanical kinetic domain} \end{array}$$

The structure matrix is block-diagonal.

The coupling occurs through the Hamiltonian which is not separated

$$H_0(x) = \frac{1}{2\,L\,(x_2)} {x_1}^2 + \frac{1}{2m} {x_3}^2 - mgx_2$$

The levitating ball: open-loop equilibria

The equilibria are given by $(x^*,\ u^*)$ such that:

$$(J-R)\,\frac{\partial H}{\partial x}(x^*) + g\,u^* = \begin{pmatrix} R\,\frac{x_1^*}{L(x_2^*)} + u^* \\ \frac{x_3^*}{m} \\ -\frac{1}{2}\,\frac{dL}{dx_2}\left(x_2^*\right)\left(\frac{x_1^*}{L(x_2^*)}\right)^2 - m\,g \end{pmatrix} = 0$$

and may be parametrized by the current: $y^* = \frac{\partial H}{\partial x_1}(x^*) = i^*$:

$$\begin{pmatrix} x_1^* \\ x_2^* \\ x_3^* \end{pmatrix} = \begin{pmatrix} L_\infty y^* + \sqrt{2mg} \\ -a + \sqrt{\frac{k}{2mg}}\quad y^* \\ 0 \end{pmatrix} \quad \text{and} \quad u^* = R\,y^*$$

they are unstable (see linearised system).

The levitating ball: matching equation

Choice of structure matrices and associated added Hamiltonian $H_a(x)$:

1. choose added structure matrices:

$$J_a(x) = \begin{pmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ -\alpha & 0 & 0 \end{pmatrix} \quad \text{and} \quad R_a(x) = 0_3$$

What's the physical interpretation?

2. Use the left annihilator $g^{\perp}(x) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, establish the PDE in H_a :

$$-\begin{pmatrix} 0 & 0 & 0 \\ -\alpha & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial H_0}{\partial x_1} \\ \frac{\partial H_0}{\partial x_2} \\ \frac{\partial H_0}{\partial x_3} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ -\alpha & -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial H_a}{\partial x_1} \\ \frac{\partial H_a}{\partial x_2} \\ \frac{\partial H_a}{\partial x_3} \end{pmatrix}$$

The levitating ball: added Hamiltonian

From : $\frac{\partial H_a}{\partial x_3} = 0$ the added potential is a function $H_a(x_1,\, x_2)$ which does not depend on the velocity and satisfies :

$$\alpha \frac{\partial H_0}{\partial x_1} = -\alpha \frac{\partial H_a}{\partial x_1} - \frac{\partial H_a}{\partial x_2}$$

with solution:

$$H_a(x_1,\, x_2) = -\int_0^{x_1} \frac{\chi}{L\left(x_2 - \frac{(\chi - x_1)}{\alpha}\right)} d\chi + \Phi\left(x_2 - \frac{x_1}{\alpha}\right)$$

For instance, if $L(x_2) = \frac{k}{(x_2 + a)}$ then
$H_a(x_1,\, x_2) = \frac{1}{2k}\left(\frac{x_1^3}{3\alpha} - x_1^2 (x_2 + a)\right) + \Phi\left(x_2 - \frac{x_1}{\alpha}\right)$

The levitating ball: IDA-PBC control

The control law is given by :

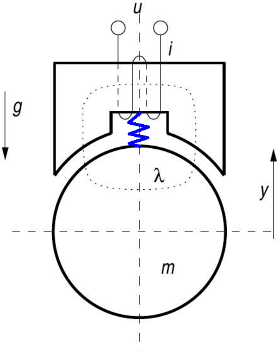
$$\beta(x) = \underbrace{1 \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}}_{=[g^t g]^{-1} g^t} \left[\begin{bmatrix} -R & 0 & \alpha \\ 0 & 0 & 1 \\ -\alpha & -1 & 0 \end{bmatrix} \frac{\partial H_a}{\partial x}(x) + \begin{pmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ -\alpha & 0 & 0 \end{pmatrix} \frac{\partial H_0}{\partial x}(x) \right]$$

or :

$$\beta(x) = -R \frac{\partial H_a}{\partial x_1}(x) + \alpha \frac{\partial H_a}{\partial x_3}(x) + \alpha \frac{\partial H_0}{\partial x_3}(x)$$

The levitating iron ball: the closed-loop system

$$\frac{dx}{dt} = \left(\underbrace{\begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 1 \\ -\alpha & -1 & 0 \end{bmatrix}}_{J} - \underbrace{\begin{bmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{R} \right) \frac{\partial H_d}{\partial x}(x)$$



Remarks

Hamiltonian methods for the control of physical systems:

1. use the interconnection of Port Hamiltonian systems and Casimir functions
2. assign closed-loop Hamiltonian function and structure matrices.

A control synthesis based on insight of desired physical behaviour in closed-loop:

1. design directly interconnection of the system with environment and indirectly the controller
2. design the closed-loop port Hamiltonian behaviour and deduce the controller

Open questions:

1. choice of desired behaviour in closed-loop and relation with performance and robustness
2. parametrization of the matching equations and solution

Concluding remarks

- Energy based modelling: based on the universal concept of energy transfer.
- Provides physical interpretation to the models and the solutions.
- Passivity is naturally encountered when working with problems arising from physical applications.
- Port-Hamiltonian control systems defines a class of non-linear passive systems which encompasses a large class of physical applications.
- A modelling and control approach which is transversal to different (or combination of) physical domains.