On the passivity based control of irreversible processes: a port-Hamiltonian approach

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Irreversible port Hamiltonian systems
Port Hamiltonian systems

PHS are suited for modelling and control of (electro) - mechanical systems

[Maschke and van der Schaft, 1992, van der Schaft, 2000]

Port Hamiltonian systems (PHS)

\[
\dot{x} = J(x) \frac{\partial U}{\partial x}(x) + g(x)u(t), \quad y = g(x)^\top \frac{\partial U}{\partial x}(x).
\]

\(x \in \mathbb{R}^n, \ U : \mathbb{R}^n \to \mathbb{R}\) the Hamiltonian function, \(J(x) \in \mathbb{R}^n \times \mathbb{R}^n\) skew-symmetric structure matrix, \(g(x) \in \mathbb{R}^m \times \mathbb{R}^n\) the input map and \(u(t) \in \mathbb{R}^m\) is a time dependent input.

Balance equations expressed by PHS

- Conservation of the Hamiltonian and of Casimir’s of the Poisson bracket

\[
\frac{dU}{dt} = \frac{\partial U}{\partial x}^\top gu = u^\top y, \quad \frac{dC}{dt} = \frac{\partial C}{\partial x}^\top gu = u^\top y
\]

- The Poisson bracket:

\[
\{Z, G\}_J = \frac{\partial Z}{\partial x}(x)J(x)\frac{\partial G}{\partial x}(x).
\]
Conservation of energy and irreversible entropy production

Many efforts to express the conservation of energy and production of entropy (GENERIC, quasi Hamiltonian systems, Brayton-Mooser formulation,..)

Simultaneous conservation of energy and irreversible entropy production

Consider a closed system:

\[ \frac{dU}{dt} = 0 \quad \text{and} \quad \frac{dS}{dt} = \sigma \left( x, \frac{\partial U}{\partial x} \right) \geq 0 \]

The second principle of thermodynamics requires \( J \) to explicitly depend on \( \frac{\partial U}{\partial x} \)

\[ \frac{\partial S}{\partial x} \top J \left( x, \frac{\partial U}{\partial x} \right) \frac{\partial U}{\partial x} = \sigma \geq 0, \quad \text{for any } U(x) \]

This is the reason to consider quasi Hamiltonian system

Retain much of the PHS structure, but their structure matrices depend explicitly on the gradient of the Hamiltonian.
Irreversible port Hamiltonian systems

\[
\dot{x} = R \left( x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x} \right) J \frac{\partial U}{\partial x} (x) + \sum_{i=1}^{m} g_i \left( x, \frac{\partial U}{\partial x} \right) u_i,
\]

i. \( x \in \mathbb{R}^n \), and \( U(x), S(x) : \mathbb{R}^n \to \mathbb{R} \) relates to the energy and entropy respectively.

ii. The constant structure matrix \( J = -J^\top \in \mathbb{R}^n \times \mathbb{R}^n \).

iii. \( gu \in \mathbb{R}^n \) describes the interaction with the environment.

with

\[
R \left( x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x} \right) = \gamma \left( x, \frac{\partial U}{\partial x} \right) \{S, U\}_J, \quad \gamma \geq 0.
\]

- \( \{S, U\}_J \) defines the thermodynamic driving force
- Entropy balance:

\[
\frac{\partial S}{\partial t} = \frac{\partial S}{\partial x}^\top R J \frac{\partial U}{\partial x} + \frac{\partial U}{\partial x}^\top gu = R\{S, U\}_J + \frac{\partial U}{\partial x}^\top gu
\]

\[
= \sigma + \frac{\partial U}{\partial x}^\top gu \quad \sigma = \gamma \left( x, \frac{\partial U}{\partial x} \right) \{S, U\}_J^2 \geq 0
\]
The heat exchanger: interaction through two conducting walls

Consider two simple thermodynamic systems with entropy balance equations

$$\begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \end{bmatrix} = \lambda \begin{bmatrix} T_2(S_2) - T_1(S_1) \\ T_1(S_1) - T_2(S_2) \end{bmatrix} + \lambda e \begin{bmatrix} 0 \\ u(t) - T_2(S_2) \end{bmatrix}$$

with $x = [S_1, S_2]^\top$ the entropies, $U = U_1(x_1) + U_2(x_2)$ the internal energy, $\frac{\partial U}{\partial x_i} = T_i(x_i) = T_0 \exp \left( \frac{x_i}{c_i} \right)$ where $T_0$ and $c_i$ are constants, $u(t)$ external heat source and $\lambda, \lambda e > 0$ Fourier’s heat conduction coefficients of the wall.

The process
The heat exchanger: interaction through two conducting walls

Consider two simple thermodynamic systems with entropy balance equations

\[
\begin{bmatrix}
\dot{S}_1 \\
\dot{S}_2
\end{bmatrix}
= \lambda
\begin{bmatrix}
\frac{T_2(S_2) - T_1(S_1)}{T_1(S_1) - T_2(S_2)} \\
\frac{T_1(S_1) - T_2(S_2)}{T_2(S_2)}
\end{bmatrix}
+ \lambda_e
\begin{bmatrix}
0 \\
\frac{u(t) - T_2(S_2)}{T_2(S_2)}
\end{bmatrix}
\]

with \( x = [S_1, S_2]^\top \) the entropies, \( U = U_1(x_1) + U_2(x_2) \) the internal energy, \( \frac{\partial U}{\partial x_i} = T_i(x_i) = T_0 \exp\left(\frac{x_i}{c_i}\right) \) where \( T_0 \) and \( c_i \) are constants, \( u(t) \) external heat source and \( \lambda, \lambda_e > 0 \) Fourier’s heat conduction coefficients of the wall.

The process may be written as IPHS

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
= \lambda
\begin{bmatrix}
\frac{1}{\partial U/\partial x_2} - \frac{1}{\partial U/\partial x_1}
\end{bmatrix}
\begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
\partial U/\partial x_1 \\
\partial U/\partial x_2
\end{bmatrix}
+ \lambda_e
\begin{bmatrix}
0 \\
\frac{1}{\partial U/\partial x_2} - \frac{1}{u}
\end{bmatrix}
\]

\( \{S, U\}_J \) is the thermodynamic driving force

\[
\{S, U\}_J = T_1 - T_2 = \frac{\partial S}{\partial x}^\top J \frac{\partial U}{\partial x} = \left[ \frac{1}{1} \right]^\top \begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix} \begin{bmatrix}
T_1 \\
T_2
\end{bmatrix}
\]

Hence, \( R(x, T) = \lambda \left( \frac{1}{T_2} - \frac{1}{T_1} \right) = \frac{\lambda}{T_1 T_2} \left( T_1 - T_2 \right) \), with \( \gamma > 0 \)
Example: chemical reactions

Consider a single reaction in a CSTR with reversible reaction scheme

\[ \sum_{i=1}^{m} \alpha_i A_i \underset{r}{\iff} \sum_{i=1}^{m} \beta_i A_i \]

with \( \alpha_i, \beta_i \) the stoichiometric coefficients for species \( A_i \). The mass balance is

\[ \dot{n}_i = r_i V + F_{ei} - F_{si} \quad i = 1, \ldots, m \]

- \( n_i \) is the number of moles of the species \( i \), \( n = (n_1, \ldots, n_m) \) \( ^\top \),
- \( r_i = \bar{\nu}_i r \), where \( r(n, T) = (r_f - r_b) \) is the reaction rate
- \( \bar{\nu}_i \) is the signed stoichiometric coefficient: \( \bar{\nu}_i = \alpha_i - \beta_i \),
- \( F_{ei} \) and \( F_{si} \) are respectively the inlet and outlet molar flows, \( F_e = (F_{e1}, \ldots, F_{em}) \) \( ^\top \)
- The volume \( V \) in the reactor is assumed to be constant as well as the pressure

The mass balance can be represented as

\[ \dot{n} = CrV + F_e - F_i \]

where \( C \) is a \( m \times 1 \) is called the stoichiometric vector.
Example: the CSTR

The chemical reaction is denoted by

\[ \nu_1 A_1 + \ldots + \nu_{m-1} A_{m-1} \rightleftharpoons \nu_m A_m, \]

\[ \nu_1, \ldots, \nu_m : \text{stoichiometric coefficients} \]

\[ A_1, \ldots, A_m : \text{chemical species} \]

together with the definition of the reaction rate:

\[ r(A, T) = r_f(A_f, T) - r_r(A_f, T) \]

with \( A \) the affinity of reaction.

The mathematical model

The balance equations [Aris, 1989],

\[ \dot{n}_i = F_{ei} - F_{si} + r_i V, \]

\[ \dot{S} = \sum_{i=1}^{m} (F_{ei} s_{ei} - F_{si} s_i) + \frac{Q}{T_e} + \sigma, \]

with \( n_i \) number of moles, \( V \) volume, \( F_{ei}, F_{si} \) molar flows, \( s_{ei}, s_i \) molar entropies, \( T_w \) jacket temperature and \( Q \) the heat flux.
Example: the CSTR

The chemical reaction is denoted by

$$\nu_1 A_1 + \ldots + \nu_{m-1} A_{m-1} \rightleftharpoons \nu_m A_m,$$

where \(\nu_1, \ldots, \nu_m\) are stoichiometric coefficients, and \(A_1, \ldots, A_m\) are chemical species.

together with the definition of the reaction rate:

$$r(A, T) = r_f(A_f, T) - r_r(A_f, T)$$

with \(A\) the affinity of reaction.

The CSTR may be expressed as IPHS with \(U\) = the internal energy

$$J = \begin{bmatrix} 0 & \ldots & 0 & \tilde{\nu}_1 \\ 0 & \ldots & 0 & \vdots \\ 0 & \ldots & 0 & \tilde{\nu}_m \\ -\tilde{\nu}_1 & \ldots & -\tilde{\nu}_m & 0 \end{bmatrix},$$

\(\tilde{\nu}_i\) stoichiometric matrix,

$$g_1 = \begin{bmatrix} n_e - n \\ \phi(x, \frac{\partial U}{\partial x}) \end{bmatrix},$$

Mass transfer,

$$g_2 = \begin{bmatrix} 0 \\ \frac{1}{T} \\ 0 \\ 1 \end{bmatrix},$$

Heat transfer,

\(\{S, U\}_J = \mathcal{A} = -\sum_{i=1}^{m} \tilde{\nu}_i \mu_i\)

\(\gamma = \frac{rV}{TA} \geq 0,\)

\(\mu_1, \ldots, \mu_m\) chemical potentials,

\(rV\) molar flow
Chemical reaction network

Consider a chemical reaction network involving $m$ chemical species, among which $m_r$ chemical reactions

$$
\sum_{i=1}^{m} \alpha_{ij} A_i \rightleftharpoons \sum_{i=1}^{m} \beta_{ij} A_i, \quad j = 1, \ldots, m_r.
$$

The basic structure underlying the dynamics of the vector $n$ of mole numbers of the chemical species is given by the **mass balance law**:

$$
\dot{n} = CrV + F_e - F_s,
$$

where the $m \times m_r$ matrix $C$ is called the **stoichiometric matrix** and whose columns are the stoichiometric vectors of each reaction: $C = [C_1, C_2, \ldots, C_{m_r}]$, and $r = [r_1, r_2, \ldots, r_{m_r}]^\top$ is the vector whose elements are the reaction rates of each individual reaction.

The energy and entropy balance are

$$
\dot{U} = U_{in} - U_{out},
$$

$$
\dot{S} = S_{in} - S_{out} + \sigma
$$
Consider the chemical reaction network
\[
\dot{n} = CrV + F_e - F_s,
\]
\[
\dot{S} = \sigma + S_{in} - S_{out},
\]
Define a vector containing the non-linear $R_j$ functions of each reaction:
\[
R \in \mathbb{R}^{m_r} = [R_1, \ldots, R_m]^T,
\]
The chemical reaction network is formulated as IPHS
\[
\dot{x} = \begin{bmatrix}
0_m \\
- R^T C^T
\end{bmatrix} \frac{\partial U}{\partial x} + \begin{bmatrix}
F_e - F_s \\
S_{in} - S_{out}
\end{bmatrix}
\]
with $x = [n_1, \ldots, n_m, S]$ the state vector, $U(x)$ and $S(x)$ respectively the internal energy and the total entropy of the complete reaction network.

Since $J_R = -J_R$ it’s straightforward to verify that the total energy is conserved. The entropy balance is
\[
\dot{S} = \frac{\partial S}{\partial x} \dot{x} = \frac{\partial S}{\partial x} J_R \frac{\partial U}{\partial x} = -R^T C^T \mu = \sum_{i=1}^{m_r} \sigma_i
\]
where $\sigma_i$ is the entropy production due to the $i$-th chemical reaction.
IPHS of chemical reaction networks

Alternatively, since the temperature in the reactor is common to all reactions, every individual (closed) reaction can be represented by the vector field

\[ X_j = R_j J_j(x) \frac{\partial U}{\partial x}(x), \]

with \( x \) the state vector of the complete network, \( J_j(x) \) the structure matrix and \( R_j \) the non-linear function containing the reaction rate of the \( j \)-th reaction.

The dynamic of the complete reaction is

\[ \dot{x} = \sum_{i=1}^{m_k} X_j + g(x, u) = \left( \sum_{i=1}^{m_r} R_i J_i \right) J_R \frac{\partial U}{\partial x} + g(x, u). \]

To verify that \( \sum_{i=1}^{m_r} R_i J_i = J_R \) it suffices to notice that the term \( R_i J_i \) can equivalently be written as

\[ R_i J_i = \begin{bmatrix} 0_m & C_i R_i \\ -R_i C_i^T & 0 \end{bmatrix}, \]

with \( C_i \) the \( m \times 1 \) stoichiometric vector.
Energy shaping of IPHS
Energy shaping of IPHS

The availability function

The “classical approach” uses the total entropy as convex extension to construct the availability function using thermodynamic considerations


Consider Gibb’s relation

\[ dU = TdS - PdV + \sum_{i=1}^{N-2} \mu_idn_i = w(z)^{\top}dz. \]

With \( z = [S, V, n_1, \ldots, n_{N-2}]^{\top} \) and \( w(z) = [T(z), -P(z), \mu_1(z), \ldots, \mu_{N-2}(z)] \) the vectors of extensive and intensive. \( U \) is a homogeneous function of degree 1, so from Euler’s Theorem,

\[ U = w(z)^{\top}z, \quad w(z) = \frac{\partial U}{\partial z}(z) \]

The energy based availability function

\[ A(x, x^*) = U(x) - \left[ U(x^*) + \frac{\partial U}{\partial x}(x^*)^\top(x - x^*) \right] \geq 0 \]

where \( x^* \) is a reference and possibly a desired steady state.
Stabilization condition

Proposition

The closed-loop equilibrium \( x^* \) of a controlled IPHS is asymptotically stable if \( A \) is strictly convex and \( x^* \) satisfies:

\[
\gamma \{ S, U \} J \{ A, U \} J + \tilde{y}^\top u = -s,
\]

\[
\tilde{y} = g^\top \frac{\partial A}{\partial x}.
\]

where \( s > 0 \) and with strict equality only at \( x^*, u^* \), where \( u^* \) is the steady state value of the control-input at the desired equilibrium.

\[
\frac{dA}{dt} = \left( \frac{\partial U}{\partial x}(x) - \frac{\partial U}{\partial x}(x^*) \right)^\top \frac{dx}{dt}
\]

\[
= -R \left( \frac{\partial U}{\partial x}(x^*)^\top J \frac{\partial U}{\partial x}(x) \right) + \left( \frac{\partial U}{\partial x}(x) - \frac{\partial U}{\partial x}(x^*) \right)^\top gu,
\]

Def. IPHS

\[
= \gamma \{ S, U \} J \{ A, U \} J + \tilde{y}^\top u,
\]

Def. Poisson Br

Since \( \gamma > 0 \) and the availability \( A \) qualifies as a Lyapunov function, the system is stable if the relation holds. Asymptotic stability follows invoking La Salle’s invariance principle on a region around \( x^* \).
Interpretation in terms of energy shaping

The closed-loop energy may be written as

\[ A(x, x^*) = U(x) + U_a(x, x^*) \]

with

\[ U_a(x, x^*) = -U(x^*) - \frac{\partial U}{\partial x}(x^*)^\top (x - x^*) \]

and \( x^* \) a desired equilibrium.

The stabilizing condition energy shaping controller. The existence of \( U_a \) implies the existence of a state feedback \( u = \beta(x, x^*) \) such that

\[ \beta(x, x^*)^\top y(x) = -\dot{U}_a(x, x^*) - s(x). \]

The closed-loop energy function \( A \) is defined by construction, hence we obtain

\[ \beta(x, x^*)^\top y(x) = \frac{\partial U}{\partial x}(x^*)^\top \dot{x} - s(x), \]

from where obtain the matching equation.
Interconnection and entropy assignment
The idea

We look for the conditions for deriving a stabilizing state feedback which renders the closed-loop system in the form

\[ \dot{x} = -\mathcal{M} \left( x, \frac{\partial U}{\partial x}, \frac{\partial A}{\partial x} \right) \frac{\partial A}{\partial x}(x), \]

with \( \mathcal{M} \left( x, \frac{\partial U}{\partial x}, \frac{\partial A}{\partial x} \right) > 0. \)

The time variation of \( A \) is in this case

\[ \dot{A} = -\frac{\partial A^\top}{\partial x}(x)\mathcal{M} \left( x, \frac{\partial U}{\partial x}, \frac{\partial A}{\partial x} \right) \frac{\partial A}{\partial x}(x) \leq 0 \]

and under some additional properness conditions the closed-loop system becomes asymptotically stable.
Define the control input as $u = \gamma \{ S, U \} J \tilde{u}$, with $\tilde{u}$ a new control input, to obtain

$$
\gamma \{ S, U \} J \{ A, U \} J + \gamma \{ S, U \} J \tilde{y}^T \tilde{u} \leq 0
$$

$$
\gamma \{ S, U \} J \left( \{ A, U \} J + \tilde{y}^T \tilde{u} \right) \leq 0
$$

In the case of reversible Hamiltonian systems $R = 1$ and $\{ A, U \} J + \tilde{y}^T \tilde{u} \leq 0$ expresses the decrease of the closed-loop Hamiltonian function chosen as candidate Lyapunov function. For IPHS $R = \gamma \{ S, U \} J$, which is not signed defined.

A particular solution is

$$
\tilde{y}^T \tilde{u} = -\{ A, U \} J - \{ S, U \} J [A, A] M + \{ A, A \} J'
$$

where $J'(x) = -J'^T(x)$ defines a Poisson bracket, and $M(x) = M^T(x) > 0$ defines a pseudo-Riemannian metric expressed by the Ginzburg-Landau dissipative bracket,

$$
[Z, G]_M = \frac{\partial Z}{\partial x}^T(x) M(x) \frac{\partial G}{\partial x}(x).
$$
Using the definition of $\tilde{y}$ and noting that $\frac{\partial A}{\partial x}^\top$ multiplies by the left, a possible control law satisfies

$$g\tilde{u} = \left(-J \frac{\partial U}{\partial x} - \{S, U\}J^\top M \frac{\partial A}{\partial x} + J' \frac{\partial A}{\partial x}\right).$$

The linear least square solution with respect to the Euclidean norm of this equation considering $\tilde{u} = \beta(x)$ is given by

$$\tilde{u} = \beta(x) = g^\dagger(x) \left(-\{S, U\}J^\top M(x) + J'(x)\right) \left(\frac{\partial U}{\partial x}(x) - \frac{\partial U}{\partial x}(x^*)\right) - g^\dagger(x)J \frac{\partial U}{\partial x}(x),$$

which exists if and only if

$$-g^\perp(x)\{S, U\}J^\top M(x) \left(\frac{\partial U}{\partial x}(x) - \frac{\partial U}{\partial x}(x^*)\right) + g^\perp(x) \left((J' - J) \frac{\partial U}{\partial x}(x) - J' \frac{\partial U}{\partial x}(x^*)\right)$$

with $g^\dagger(x) = [g^\top(x)g(x)]^{-1}g^\top(x)$ the Moore-Penrose pseudo inverse and $g^\perp(x)$ a left full rank annihilator of $g(x)$, i.e., $g^\perp(x)g(x) = 0$. These equations express a class of IDA-PBC problem.
Let us compare with the solution of a *reversible* port-Hamiltonian system

\[ \dot{x} = J \frac{\partial U}{\partial x} + g \ddot{u} \]

then the same target system becomes

\[ \dot{x} = \left( -\{S, U\}JM(x) + J'(x) \right) \frac{\partial A}{\partial x}, \]

where the matrix \( \{S, U\}JM(x) \) is not necessary positive semi-definite.

This “contradiction” comes from the definition of the control input, \( u = \gamma \{S, U\} \ddot{u} \), but it also expresses the irreversible nature of the control system. It has been reported in the literature as the “dissipation obstacle” for reversible PHS and as the “interconnection obstacle” for IPHS.
Proposition

Assume $A$ is strictly convex and that there exist matrices $M(x) \geq 0$ and $J'(x) = -J'^\top(x)$, and a full-rank left annihilator $g^\perp(x)$ of $g(x)$ that verify the matching equation. Then the control $u = \gamma \{S, U\} \beta(x)$ asymptotically stabilizes the closed-loop equilibrium $x^*$. Furthermore, the closed-loop system is given by the pseudo-PHS

$$\dot{x} = \left( -\sigma M(x) + RJ'(x) \right) \frac{\partial A}{\partial x}.$$ 

In some cases it is possible to use the physical properties of the system to solve the matching equation.

Corollary

Assume $J' = J$, then the closed-loop equilibrium $x^*$ is asymptotically stable if

$$g^\perp(x) J \frac{\partial U}{\partial x}(x^*) = 0,$$

and

$$g^\perp(x) M(x) = 0.$$
Example: the CSTR

Let us select $J' = J$ and find and leave the matrix $M$ as degree of freedom. One possible annihilator for $g$ is

$$
 g^\perp(x) = \begin{bmatrix}
 \tilde{n}_2 & -\tilde{n}_1 & 0 & \ldots & 0 & 0 & 0 \\
 0 & \tilde{n}_3 & -\tilde{n}_2 & \ldots & 0 & 0 & 0 \\
 0 & 0 & \ddots & \ddots & \ddots & 0 & 0 \\
 0 & 0 & \ldots & 0 & \tilde{n}_m & -\tilde{n}_{m-1} & 0 
\end{bmatrix}
$$

Hence, for the first condition of the Corollary we obtain

$$
 g^\perp(x)J = \begin{bmatrix}
 0 & \ldots & 0 & \nu_1 \tilde{n}_2 - \nu_2 \tilde{n}_1 \\
 0 & \ldots & 0 & \nu_2 \tilde{n}_3 - \nu_3 \tilde{n}_2 \\
 \ddots & \ddots & \ddots & \ddots \\
 0 & \ldots & 0 & \nu_{m-1} \tilde{n}_m - \nu_m \tilde{n}_{m-1}
\end{bmatrix}.
$$

It suffices to verify that $g^\perp(x)J = 0$, which is true if

$$
 \frac{\tilde{n}_1}{\nu_1} = \frac{\tilde{n}_2}{\nu_2} = \cdots = \frac{\tilde{n}_{m-1}}{\nu_{m-1}} = \frac{\tilde{n}_m}{\nu_m}.
$$

From the assumption of constant volume the initial numbers of moles of each specie equals the numbers of moles at the inlet, i.e., $n(t = 0) = n_0 = n_e$. Hence,

$$
 \frac{n_{0i} - n_i}{\nu_i} = \xi, \quad \text{De Donder’s extent of reaction} \rightarrow \text{(always holds)}
$$
The second condition of the Corollary can be solved noting that the last column of $g^\perp(x)$ is zero. This implies that any matrix $M(x) = M^\top(x) > 0$ for which the first $m$ rows and columns form a null submatrix is solution. A simple choice is $M = \text{diag}(0, \ldots, 0, 1)$.

The closed-loop system takes the form

$$
\dot{x} = \left(-\sigma M + RJ\right) \frac{\partial A}{\partial x}(x, x^*)
$$

The dynamic of the entropy is given by

$$
\dot{S} = -\sum_{i=1}^{m} \mu_i(x) \nu_i \frac{r}{T} + \left[\sum_{i=1}^{m} \mu_i(x^*) \nu_i \frac{r}{T} - k(x)\sigma(T - T^*)\right]
$$

The time derivative of $A$ is

$$
\frac{dA}{dt} = -\sigma (T - T^*)^2.
$$

The dissipation of the availability function is related to the irreversible energy production, which is positive definite and only zero at the thermodynamic equilibrium (which is not included in the region of operation of the CSTR).
Final remarks and future work
### Final remarks

- The **energy based availability function** has been combined with the IPHS framework to derive an asymptotic stability condition.
- The stability condition has been interpreted in terms of the **matching of vector fields**, leading to a class of **IDA-PBC** for irreversible thermodynamic systems.
- The matching condition and the definition of the control clearly express the **irreversible nature** of the system and it has been shown that the closed-loop system is an **dissipative pseudo PHS** with the energy based availability as Hamiltonian function.
- A synthesis method for IPHS which permit to shape the closed-loop energy, interconnection structure and entropy creation has been proposed and illustrated on a general model of a **CSTR**.

### Future work

- IPHS for infinite dimensional systems (tubular reactor)
- Stabilization of reaction networks
Thanks!

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